LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

FIFTH SEMESTER – **APRIL 2024**

UST 5501 – APPLIED STOCHASTIC PROCESSES

Date: 02-04-2024   Dept. No.   Max. : 100 Marks
Time: 09:00 AM - 12:00 NOON

SECTION –A

Answer ALL the questions.   **10 x 2 = 20 Marks**

1. Define state space and index parameter.

2. Define n-step transition probability.

3. When a state is said to be transient?

4. Give an example for reducible Markov chain.

5. Write two examples for Poisson process.

6. Provide the postulates for birth and death process.

7. Define renewal process.

8. State any two properties of periodicity.

9. Define branching process.

10. Write a note on extinction probability.

SECTION –B

Answer any FOUR questions .   **4 x 10 = 40 Marks**

11. Explain Spatially homogeneous Markov chains.

12. Prove that the one-dimensional random walk is recurrent.

13. Consider the following one-step transition probabilities:

   $P_{11} = \frac{1}{2}$, $P_{12} = \frac{1}{4}$, $P_{13} = \frac{1}{4}$, $P_{21} = 1/3$, $P_{23} = 2/3$, $P_{31} = \frac{1}{2}$, $P_{32} = \frac{1}{2}$.

   Find the stationary probability distribution.

14. Define communication of states and prove that communication is an equivalence relation.

15. Derive the differential equations for pure birth process.

16. State and prove the elementary renewal theorem.

17. Establish the PGF relation for branching process.

18. State and prove the necessary and sufficient condition for state i to be recurrent.

SECTION –C

19. State the postulates of Poisson process and derive $P_n(t)$ for it.

20. Consider the state space S = {1,2,3,4,5,6} with the following one-step transition

    probabilities:

    $P_{11}= 1/3$, $P_{13} = 2/3$, $P_{22} = 1/2$, $P_{23} = P_{25} = 1/4$, $P_{31} = 2/5$,

    $P_{33} = 3/5$, $P_{42} = P_{43} = P_{44} = P_{46} = 1/4$, $P_{55} = P_{56} = 1/2$, $P_{65} = 1/4$ and $P_{66} =3/4$.

    (a)Draw the transition diagram and form the transition matrix. (3)

    (b)Find the equivalence classes.(3)

    (c)Show that states 1,3,5 and 6 are recurrent.(10)

    (d)Prove that states 2 and 4 are transient.(4)

21. Explain in detail Type I and Type II counter models discussed in renewal process.

22. Derive mean and variance for discrete branching process.

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